Space Surveillance: towards new tools for cataloguing space debris

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Space Surveillance

Credit: European Space Agency

Why bother?

- 1,419 satellites currently orbiting Earth (as of 30/06/16)[1]
- 716 for Communications, 106 for Navigation/Positioning
- Debris (as of August 2013)[2]
  - More than 170 million of sizes larger than 1 mm
  - 670,000 of sizes larger than 1 cm
  - 29,000 of sizes larger than 10 cm
  - speeds up to 18,000 miles per hour
- Inadvertent reentry
- NASA's LDE Facility documented over 20,000 impacts
- Space maintenance costs increased by up to 26%[3]

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[1] http://www.ucsusa.org/nuclear-weapons/space-weapons/satellite-database

[2] ESA - How_many_space_debris_objects_are_currently_in_orbit

[3] Ailor et al., "Space Debris and the Cost of Space Operations," Proc. 4th IAASS. ESA-SP Vol. 680, 2010

Problem

- Determine orbits of small objects in deep space, at geostationary altitudes
- Orbit determination over different observation sessions to work around the low observability
- Which pairs of observations to process with a more accurate orbit determination algorithm?
- Requirements:
  - no need to give answers in real time
  - no tracking required
  - computationally efficient
- Proposed solution incorporates
  - a new "supplementary" orbital model
  - a new initial orbit determination method
  - a method for solving the observation-to-observation associations

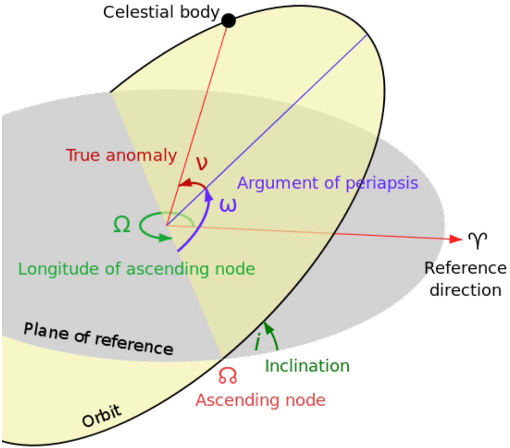
Challenges

According to [Stauch,2014]

- Large Observation-to-observation association problem
- Angles-only, short-arc statistical initial orbit determination (SIOD)
- Arc-lengths of observations magnitude similar as the measurement noise
- SIOD methods introduce large uncertainties (Constrained Admissible Region)
- Many closely-spaced objects, exacerbates the data association problem
- Need for SIOD given a single observation

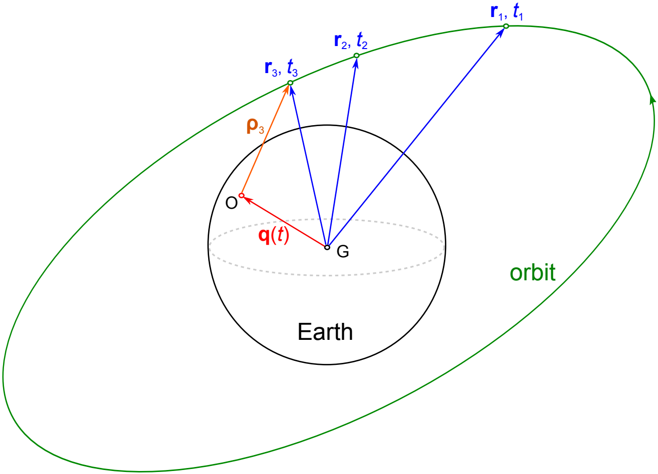
[Stauch,2014] Jason Stauch, Moriba Jah, Jason Baldwin, Thomas Kelecy, and Keric A. Hill. "Mutual Application of Joint Probabilistic Data Association, Filtering, and Smoothing Techniques for Robust Multiple Space Object Tracking (Invited)", AIAA/AAS Astrodynamics Specialist Conference, AIAA SPACE Forum, (AIAA 2014-4365)

Classical orbital elements

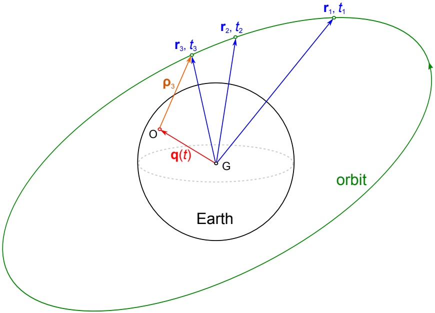


Credit: Wikipedia

Initial orbit determination



Initial orbit determination



- $\boldsymbol{\rho}_i = \rho_i \hat{\boldsymbol{\rho}}_i = \rho_i (\cos\alpha_{t,i}\cos\delta_{t,i}, \sin\alpha_{t,i}\cos\delta_{t,i}, \sin\delta_{t,i})^T$, given topocentric measurements $\{(\alpha_{t,i}, \delta_{t,i}), t_i : i = 1, 2, 3\}$
- IOD produces first estimate $\tilde{\boldsymbol{\Theta}}_0 = (a, e, \Omega, i, \omega, \nu)^T$

Laplace's method

- Two-body motion: $\ddot{\boldsymbol{r}} = -\frac{\mu}{r^3}(\rho\hat{\boldsymbol{\rho}} + \boldsymbol{q}) = \ddot{\rho}\hat{\boldsymbol{\rho}} + 2\dot{\rho}\dot{\hat{\boldsymbol{\rho}}} + \rho\ddot{\hat{\boldsymbol{\rho}}} + \ddot{\boldsymbol{q}}$
- Interpolate $\hat{\boldsymbol{\rho}}$, $\dot{\hat{\boldsymbol{\rho}}}$ and $\ddot{\hat{\boldsymbol{\rho}}}$ on time by the Lagrange polynomial
- $\ddot{\boldsymbol{q}} = \boldsymbol{\omega}_E \times \dot{\boldsymbol{q}}$, $\dot{\boldsymbol{q}} = \boldsymbol{\omega}_E \times \boldsymbol{q}$
- Solve linear system of the form $(\rho, \dot{\rho}, \ddot{\rho})^T A_L^T = B_L^T$
- Middle position: $r_2^8 + a_L r_2^6 + b_L r_2^3 + c_L = 0$
- Middle velocity: $\dot{\boldsymbol{r}}_2 = \dot{\rho}\hat{\boldsymbol{\rho}}_2 + \rho\dot{\hat{\boldsymbol{\rho}}}_2 + \dot{\boldsymbol{q}}_2$

Overview

- Classical methods
  - Laplace
  - Gauss
  - Väisäla
- "Hybrid" (numerical/analytic) methods
  - Escobal's double-r
  - Gooding
    - Solve Lambert's problem for iterated $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_3$ and t_{13}
    - Compute sight-line for time t_2: $\hat{\boldsymbol{\rho}}_{2,c}$
    - Newton-Raphson method to minimize $\hat{\boldsymbol{\rho}}_{2,c} - \hat{\boldsymbol{\rho}}_2$
- Problems:
  - measurements are actually stochastic
  - short-arc length is within the measurement noise standard deviation
  - high sensitivity to the observation geometry and quality

Considerations and assumptions

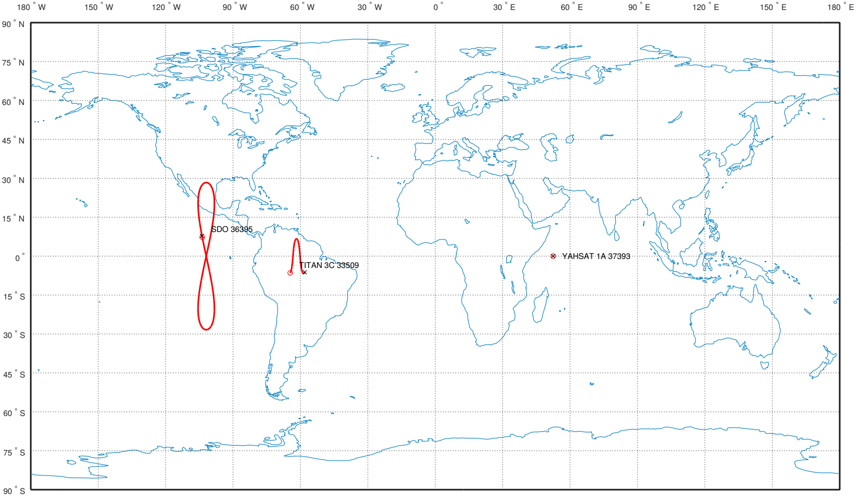
- Considerations
  - Goal of IOD: reasonable first-order estimate of orbit elements
  - Short-arc IOD: high uncertainties may conceal second-order effects on predictions
  - Stochastic treatment:
    - simple model for the mean unperturbed motion
    - process noise (perturbations and short-periodic terms)
- Modeling assumptions
  - Unperturbed Keplerian motion about a given force centre
  - Negligible orbit eccentricity ($e \approx 0$)

Model

$$\ell(t) = \omega + M_0 + n \cdot (t - t_0),$$
$$\alpha(t) \approx \Omega + \arctan\left[\tan \ell(t) \cdot \cos i\right],$$
$$\delta(t) \approx \arcsin\left[\sin \ell(t) \cdot \sin i\right],$$
$$\lambda(t) = \lambda_{GHA} + \alpha(t) - \omega_E \cdot (t - t_0),$$
$$\phi(t) = \arctan\left[(1 - e_E^2)^{-1/2} \tan \delta(t)\right];$$

- ℓ: mean unperturbed latitude
- α and δ: geocentric right ascension and declination
- λ and ϕ: longitude and geodetic latitude of the ground track
- λ_{GHA}: Greenwich hour angle
- ω_E and e_E: Earth's rotation velocity and first eccentricity
- M_o: mean anomaly at epoch t_0
- ω, i, Ω: osculating elements

Prediction


180 ° W
150 ° W
120 ° W
90 ° W
60 ° W
30 ° W
0 °
30 ° E
60 ° E
90 ° E
120 ° E
150 ° E
180 ° E
90 ° N
75 ° N
60 ° N
45 ° N
30 ° N
15 ° N
0 °
15 ° S
30 ° S
45 ° S
60 ° S
75 ° S
90 ° S
SDO 36395
TITAN 3C 33509
YAHSAT 1A 37393

STK simulation

Credit: Airbus Safran Launchers/France

Considerations

- Task eased by prior knowledge of objects' behavior (near geosynchronicity)
- Judicious choice of approximations and fixed parameters
  - disregard of possible but out-of-context solutions
- Simplicity and functionality x generality
- Computationally fast

Procedure

Set $a := R_{\text{geo}} = 35786\,\text{km}$, $n := n_{\text{geo}}$ and initialize $\boldsymbol{r}_1$ and $\boldsymbol{r}_3$
Iteration in $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_3$:

1. $\boldsymbol{r}_2 = \frac{\boldsymbol{q}_2 \cdot (\boldsymbol{r}_1 \times \boldsymbol{r}_3)}{\hat{\boldsymbol{\rho}}_2 \cdot (\boldsymbol{r}_1 \times \boldsymbol{r}_3)} \cdot \hat{\boldsymbol{\rho}}_2 + \boldsymbol{q}_2$, $\rho_2 = -\boldsymbol{r}_2 \cdot \boldsymbol{q}_2 + \sqrt{(\boldsymbol{r}_2 \cdot \boldsymbol{q}_2)^2 + r_2^2 - q_2^2}$
2. Compute p (Escobal's method or local estimate $p \approx \|\boldsymbol{h}_{13}\| / \mu$)
3. Iterate eccentricity e (Newton's method)
   - $\nu_i = \arccos\left[\frac{a}{r_i\, e\, \sqrt{1-e^2}}\right]$, for $i = 1, 2, 3$
   - $E_i = 2 \arctan\left[\frac{\sqrt{1-e^2}}{\sqrt{1+e^2}} \tan\frac{\nu_i}{2}\right]$, $M_i = E_i - e \sin E_i$
   - Minimize $\epsilon_{t,1} = t_{12} - \Delta M_{12}/n$, $\epsilon_{t,3} = t_{32} - \Delta M_{32}/n$
4. Compute (α_i, δ_i) using ρ_i, $(\alpha_{t,i}, \delta_{t,i})$ and site position [Roy,1988]
5. Iterate $\Omega_{k+1} = \Omega_k - \frac{\tan\delta_3 \sin(\alpha_1 - \Omega_k) - \tan\delta_1 \sin(\alpha_3 - \Omega_k)}{-\tan\delta_3 \cos(\alpha_1 - \Omega_k) - \tan\delta_1 \cos(\alpha_3 - \Omega_k)} + \ldots$

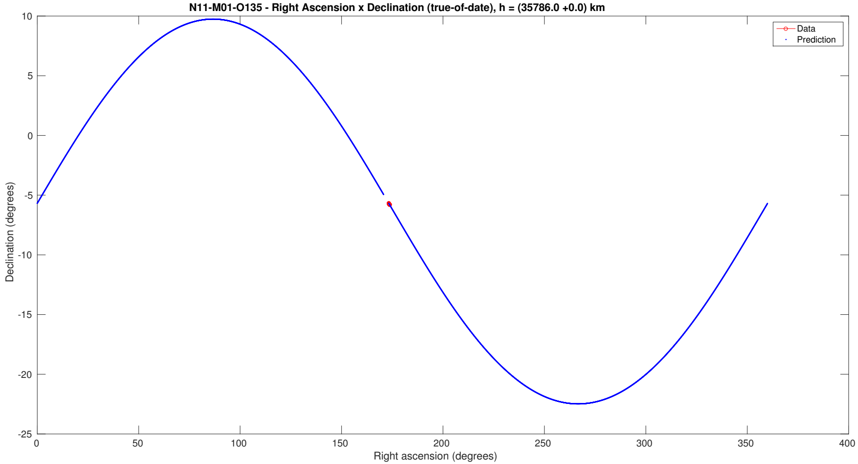
[Roy,1998] Archie E. Roy, "Orbital Motion," Taylor & Francis, 3rd ed., 1988

Procedure (cont.)

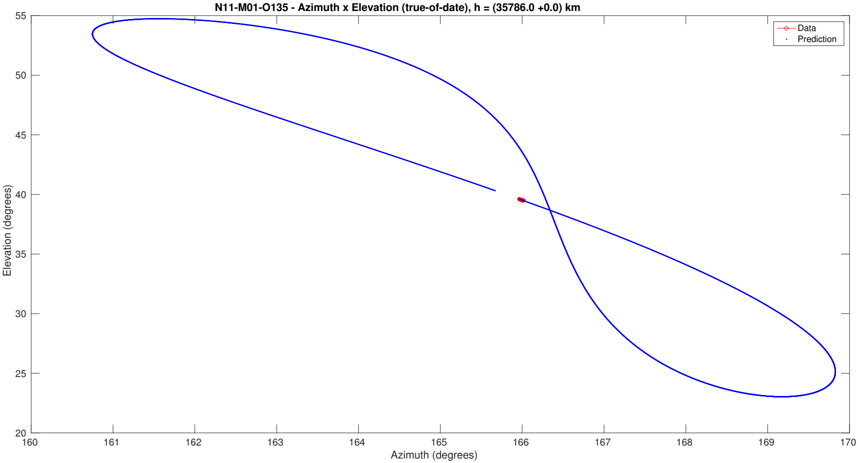
6. Inclination $I = I_2 = \arctan\left[\tan\delta_2 / \sin(\alpha_2 - \Omega)\right]$
7. Mean argument of latitude $\ell_i = \arcsin\left[\sin\delta_i / \sin I\right]$
8. Argument of perigee $\omega = \Omega - \ell_2$
9. Update $\boldsymbol{r}_i$ as function of r_i, Ω, I, ℓ_i
10. Update slant-ranges $\boldsymbol{\rho}_i = \boldsymbol{r}_i - \boldsymbol{q}_i$, and estimate topocentric $(\hat{\alpha}_{t,i}, \hat{\delta}_{t,i})$
11. Verify error $\epsilon_i = \|(\hat{\alpha}_{t,i}, \hat{\delta}_{t,i}) - (\alpha_{t,i}, \delta_{t,i})\|$ (stopping criteria)

- Refine solution by least squares of $(\hat{\alpha}_{t,i}, \hat{\delta}_{t,i}) - (\alpha_{t,i}, \delta_{t,i})$ using a "complete" model that includes e, a and n explicitly

Example


N11-M01-O135 - Right Ascension x Declination (true-of-date), h = (35786.0 +0.0) km
Data
Prediction
Declination (degrees)
Right ascension (degrees)
10
5
0
-5
-10
-15
-20
-25
0
50
100
150
200
250
300
350
400

Example

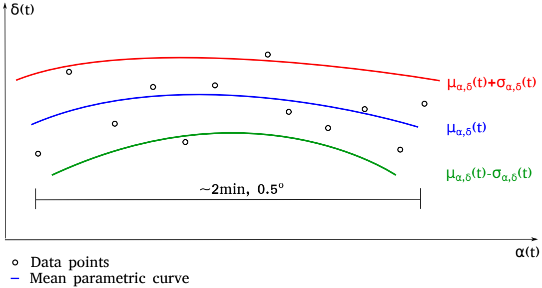

N11-M01-O135 - Azimuth x Elevation (true-of-date), h = (35786.0 +0.0) km
Data
Prediction
Elevation (degrees)
Azimuth (degrees)
55
50
45
40
35
30
25
20
160
161
162
163
164
165
166
167
168
169
170

Overview

- **Data:** campaign of observations providing multiple-session batches
- **Step 1:** IOD generates a database with, for every observed object:
  - orbital elements for mean parametric curve
  - orbital elements for parametric curves distant "one-sigma" from the mean curve
- **Step 2:** for each object, the parameters are used to propagate forward the orbit corresponding to the mean curve and its uncertainty
- **Step 3:** the same orbits propagated backwards
- **Step 4:** Gating is used to identify pairs of objects that will have significant likelihood to be associated
- **Step 5:** Pairwise association likelihood function is evaluated
- **Step 6:** Threshold likelihood function to determine the associations

Step 1: IOD and uncertainty by parametric curves

- Arc-length functional process $\{\gamma(t)\}_{t\geq t_0}$, $\gamma(t) := \int_{t_0}^{t} \sqrt{1+\left(\frac{d\delta}{d\alpha}\right)^2}\dot{\alpha}dt$[4]
- Statistics on $\gamma(t)$: $\mu_{\alpha,\delta}(t) = \mathbb{E}\left[\gamma(t)\right]$, $\Sigma_{\alpha,\delta}(t,s) = \mathrm{cov}\left(\gamma(t), \gamma(s)\right)$
- Measurement noise variance $\sigma^2_{\gamma,m}(t) = \left(\frac{\partial\gamma}{\partial\alpha}\right)^2 \sigma^2_{m,\alpha} + \left(\frac{\partial\gamma}{\partial\delta}\right)^2 \sigma^2_{m,\delta}$
- Initial uncertainty $\sigma^2_{\alpha,\delta}(t_k) = \Sigma_{\alpha,\delta}(t_k, t_k) := \sigma^2_{\gamma,m}(t_k)$, $\Sigma_{\alpha,\delta}(t_k, t_l) = 0$
- IOD translates $\sigma^2_{\alpha,\delta}(t)$ into initial covariance of orbital elements $\Sigma_{\Theta}(t_0)$



[4] alternative: $\gamma(t) := d\delta/d\alpha$

Steps 2 and 3: Forward and backward predictions

- State $\mathbf{\Theta} = (a, e, \Omega, I, \omega, M)^T$, $\boldsymbol{w} \sim \mathcal{N}(0, \mathbb{I}_6)$
- Linear state-space model

$$\mathbf{\Theta}(t_{k+1}) = \mathrm{F}_k \mathbf{\Theta}(t_k) + \mathrm{Q}_{k+1}^{1/2} \boldsymbol{w},\ \mathbf{\Theta}(t_{k-1}) = \mathrm{B}_k \mathbf{\Theta}(t_k) + \mathrm{Q}_{k-1}^{1/2} \boldsymbol{w}$$

- Unperturbed transition matrices:

$$\mathrm{F}_k = \begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & \ddots & \\ 0 & & & 1 + \Delta t \sqrt{\mu / a_k^3} \end{bmatrix},\ \mathrm{B}_k = \begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & \ddots & \\ 0 & & & 1 - \Delta t \sqrt{\mu / a_k^3} \end{bmatrix}$$

- Process noise matrix Q_k is scaled by non-secular effects and short-term periodic terms ([Vinti,1998], pages 187-191)
- Objects discriminated in the same observation session are clustered together (no comparison within cluster)
- Propagation is performed through all other sessions, to match the initial observation time of other objects

Steps 4, 5 and 6 - Likelihood evaluations

- Propagated set of orbital elements gates objects from other clusters
- N objects, ith object is represented by the random vector $\mathbf{\Theta}_i(t_i) \in \mathbb{R}^6$, with original realization at time t_i
- Association event $\alpha(i) : \{i = 0, 1, 2, \ldots, N\} \to \{1, 2, \ldots, N\} \setminus \{i\}$.
- Likelihood of a single association event is modeled as:

$$p(\alpha(i)|\boldsymbol{y}_{1:N}) \propto \begin{cases} \frac{1}{2} P_o^i \mathbb{E}_{\Theta_i(t_{\alpha(i)})} \left[p(\boldsymbol{y}_{\alpha(i)}|\mathbf{\Theta}_i(t_{\alpha(i)})) \right] & + \\ \frac{1}{2} P_o^i \mathbb{E}_{\Theta_{\alpha(i)}(t_i)} \left[p(\boldsymbol{y}_i|\mathbf{\Theta}_{\alpha(i)}(t_i)) \right], & i = 1, \ldots, N; \\ \lambda_c \left(1 - P_o^i\right), & i = 0, \end{cases}$$

- Probability that an object can be observed a second time P_o^i, and the expected clutter density λ_c
- Measurements $\boldsymbol{y}_j = (a, e, I, \alpha, \delta)_j^T$ are realizations $\boldsymbol{y}_j \sim p(\boldsymbol{y})$
- The result is a (normalized per object) likelihood table of associations
- Thresholding the likelihood table produces lists of associated objects

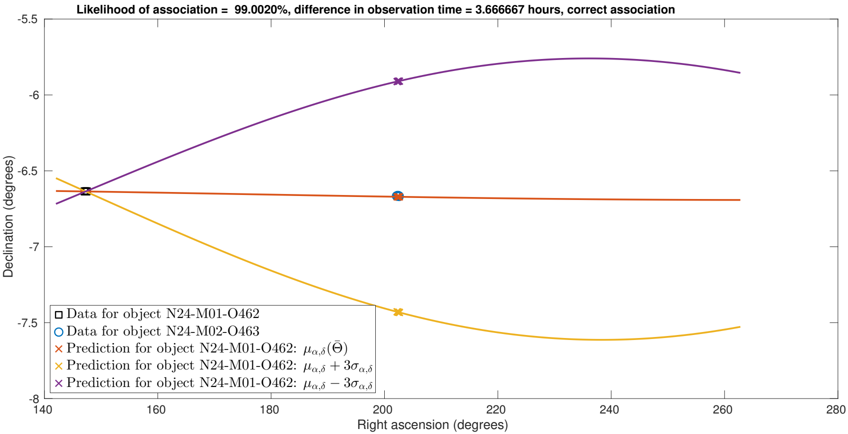
Steps 4, 5 and 6 (continuation)

$$
\begin{aligned}
\mathbb{E}_{\Theta_i(t_{\alpha(i)})}\left[p(\boldsymbol{y}_{\alpha(i)}|\boldsymbol{\Theta}_i(t_{\alpha(i)}))\right] &= \int_{\mathbb{R}^6} p(\boldsymbol{y}_{\alpha(i)}|\boldsymbol{\Theta}_i(t_{\alpha(i)}))p(\boldsymbol{\Theta}_i(t_{\alpha(i)}))d\boldsymbol{\Theta}_i(t_{\alpha(i)}), \\
\mathbb{E}_{\Theta_{\alpha(i)}(t_i)}\left[p(\boldsymbol{y}_i|\boldsymbol{\Theta}_{\alpha(i)}(t_i))\right] &= \int_{\mathbb{R}^6} p(\boldsymbol{y}_i|\boldsymbol{\Theta}_{\alpha(i)}(t_i))p(\boldsymbol{\Theta}_{\alpha(i)}(t_i))d\boldsymbol{\Theta}_{\alpha(i)}(t_i),
\end{aligned}
$$

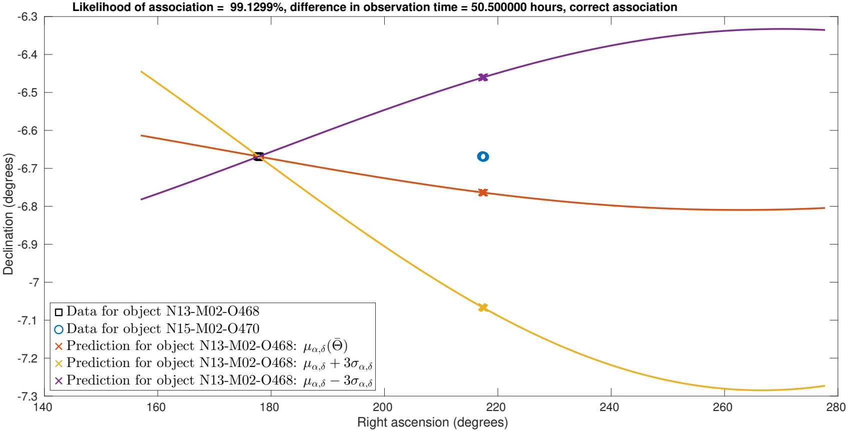
$$
\begin{aligned}
p(\boldsymbol{\Theta}_i(t_{\alpha(i)})) &= \int_{\mathbb{R}^6} p(\boldsymbol{\Theta}_i(t_{\alpha(i)})|\boldsymbol{\Theta}_i(t_i))p(\boldsymbol{\Theta}_i(t_i))d\boldsymbol{\Theta}_i(t_i), \\
p(\boldsymbol{\Theta}_{\alpha(i)}(t_i)) &= \int_{\mathbb{R}^6} p(\boldsymbol{\Theta}_{\alpha(i)}(t_i)|\boldsymbol{\Theta}_{\alpha(i)}(t_{\alpha(i)}))p(\boldsymbol{\Theta}_{\alpha(i)}(t_{\alpha(i)}))d\boldsymbol{\Theta}_{\alpha(i)}(t_{\alpha(i)}).
\end{aligned}
$$

[Vinti,1998] John Vinti, "Orbital and Celestial Mechanics," Volume 177 of Progress in astronautics and aeronautics, AIAA, 1998

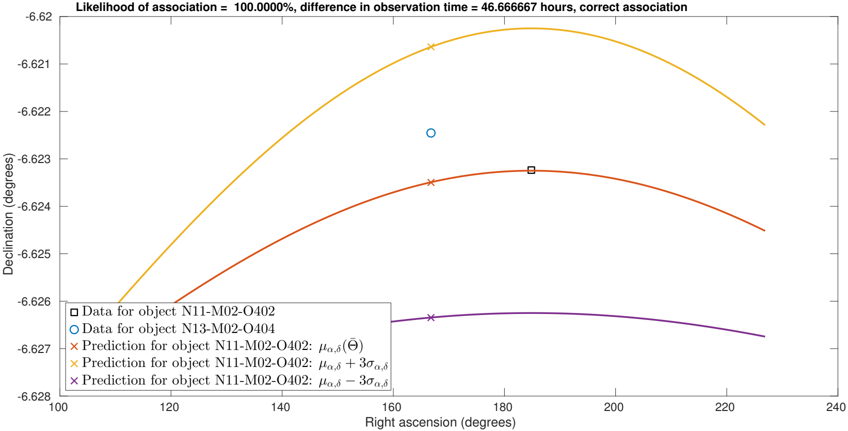
Exemplar association 1



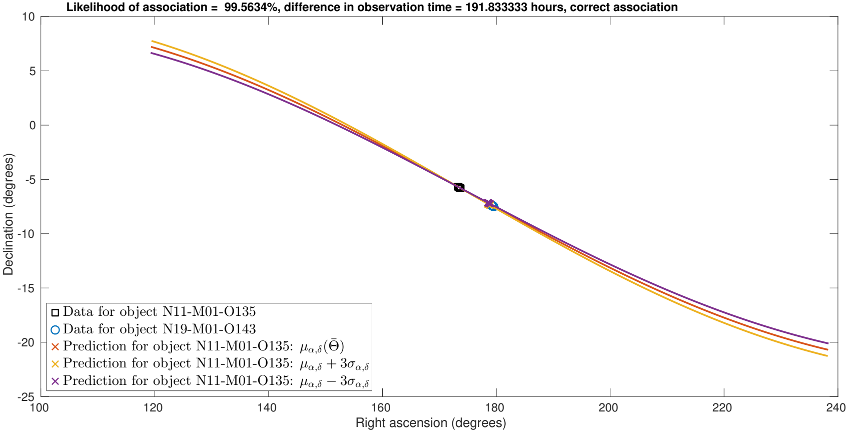
Exemplar association 2



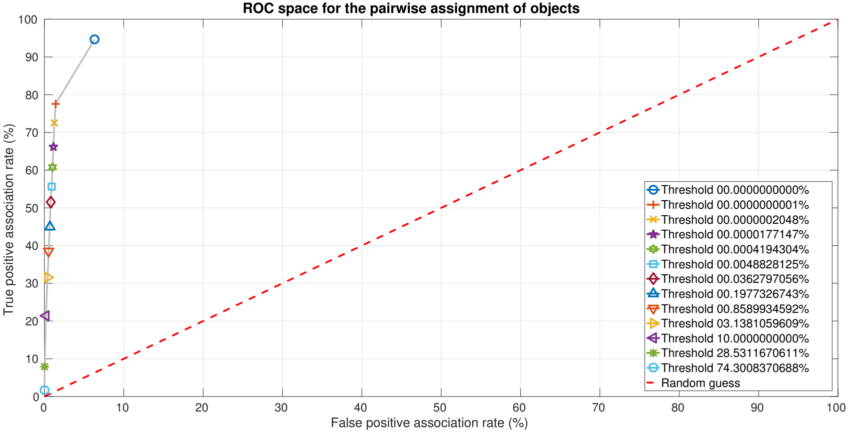
Exemplar association 3



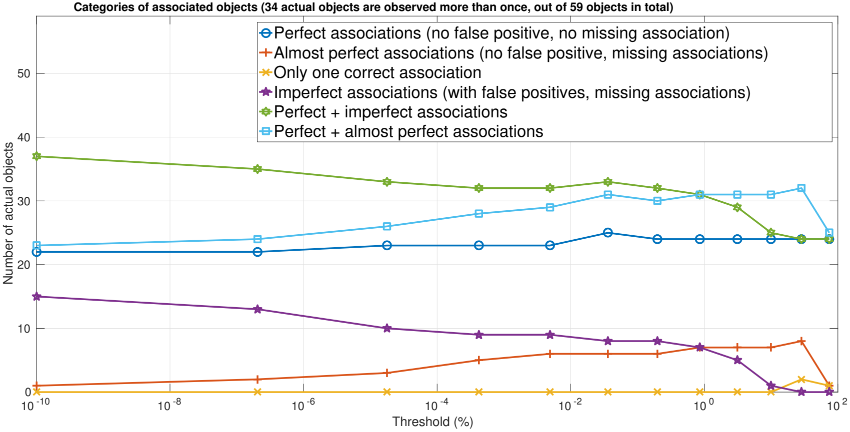
Exemplar association 4



ROC space plot for pairwise associations



Categories of associated objects



Conclusions

- Specifically for our method, tests suggest our IOD method performs better than established ones (Gauss, Laplace, Gooding)
- There is a bold assumption that non-observed objects do not exist: one could incorporate negative information (e.g., probability of existence)
- A PHD filter could capture the features of the objects in the elements' space, avoiding explicit comparison between objects
- The case made suggests that judiciously chosen assumptions and algorithm settings may lead to outstanding results, but possibly be detrimental to generality.

Future work

- The IOD method can be extended to incorporate a model of synchronous elements, or iterate slant-ranges by solving the Lambert problem
- The method is not computationally expensive: it can be made more scalable by preliminary matching of orbital elements that vary slowly with the orbit phase, and using kd-trees
- We intend to write a journal paper shortly but we welcome comments to help inform what to write

Questions